SCAF Folding Practices That Enhance MathematicsLearning

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ABSTRACT. It is over 25 years since Wood, Bruner and Ross (1976, *Journal of Child Psychology and Psychiatry, 17*, 89–100) introduced the idea of 'scaffolding' to represent the way children's learning can be supported. Despite problems, this metaphor has enduring attraction in the way it emphasises the intent to support a sound foundation with increasing independence for the learner as understanding becomes more secure. It has resonance with the widely accepted notion in teaching of construction and the constructivist paradigm for learning. The discussion that follows will characterise some teaching approaches that can be identified as scaffolding, revisiting some of the original classifications, and identifying further scaffolding strategies with particular reference to mathematics learning. Examples will be given from studies relating to geometry learning with four to 6 year olds and to arithmetic learning with older pupils.

KEY WORDS: classroom interactions, explaining, mathematics, scaffolding, teaching and learning

Mathematics teaching is informed by the social constructivist paradigm for the teaching–learning process in which 'students actively construct meaning as they participate in increasingly substantial ways in the re-enactment of established mathematical practices' (Cobb, Yackel, & McClain, 2000 p. 21). Marked changes from traditional teaching approaches are needed as the role of the teacher changes from 'showing and telling' to responsive guidance in developing pupils' own thinking. This guidance requires a range of support for pupils' thought constructions, in a way that develops individual thinking as well as leading to the generation of mathematically valid understandings. Teachers work to establish classroom practices in which patterns of instruction are established to support this learning. The notion of 'scaffolding' has been used to reflect the way adult support is adjusted as the child learns and is ultimately removed when the learner can 'stand alone' (Wood, Bruner, & Ross, 1976). The following discussion is an attempt to identify a hierarchy of interactions which relate to teaching practices that can enhance mathematics learning. Starting with a review of historic notions, it will go on to identify observed classroom practices that are classified as different levels of scaffolding.

BACKGROUND

Characterising Scaffolding

Introducing the metaphor of scaffolding to help explore the nature of adult interactions in children's learning, Wood et al. (1976) identified six key elements:

- *recruitment* – enlisting the learner's interest and adherence to the requirements of the task;
- *reduction in degrees of freedom* – simplifying the task so that feed-back is regulated to a level that could be used for correction;
- *direction maintenance* – (verbal prodder and corrector) keeping the learner in pursuit of a particular objective;
- *marking critical features* – (confirming and checking) accentuating some and interpreting discrepancies;
- *frustration control* – responding to the learner's emotional state;
- *demonstration* – or modelling solution to a task. (p. 98)

In discussing these, the authors hint at complexities that need further analysis, for example, in demonstrating or 'modelling' a solution to a task ''the tutor is 'imitating' in idealised form an attempted solution tried (or assumed to be tried) by the tutee in the expectation that the learner will then 'imitate' it back in a more appropriate form''. They go on to propose, ''the only acts children imitate are those they can already do fairly well'' (Wood et al., 1976 p. 99). This has some resonances with classroom practices and teachers will recognise the role of these supporting interactions.

Again working with the adult as leader in the learning situation, Tharpe and Gallimore (1988) use the term '*assisted learning*' to develop the classification of adult interactions and identify six interdependent strategies:

- *modelling* – offering behaviour for imitation;
- *contingency management* – rewards and punishment arranged to follow on behaviour;
- *feeding back* – information resulting from experiences;
- *instructing* – calling for specific action;
- *questioning* – calling for linguistic response;

- *cognitive structuring* – providing explanations and belief structures that organise and justify. (p. 42)

These at first appear to be different from those of Wood, Bruner and Ross but have some commonality such as *demonstration/modelling* and *frustration control/contingency management* and *marking critical features/feeding back*. However, *questioning* and *cognitive structuring* begin to suggest more of the interactions that typify good classroom exchanges. Tharpe and Gallimore (1988) suggest that *cognitive structuring*, which provides a 'structure for thinking and acting', is the most comprehensive and most 'intuitively obvious' supporting strategy. They note, however, that study after study has documented the absence in classrooms of this fundamental tool: assistance provided by more capable others that is responsive to goal-directed activities.

More recent studies propose that it is crucial to consider the role of the learner, as sociocultural factors cannot be ignored and the classroom, as a social environment, involves complex exchanges that support learning. Rogoff focuses on both the learner and the 'teacher' as being mutually dependent 'in ways that preclude their separation' (Rogoff, 1995, p. 140). She analyses the interactions between adults and children, identifying three planes of 'activities' or 'events' corresponding to personal, interpersonal and community processes. She uses the term '*Participatory appropriation*' to identify the process by which individuals transform their personal understanding. '*Guided participation*' refers to the interpersonal plane that includes face-to-face interactions andside-by-side joint participation including, but going beyond, the idea of assisted learning identified above. The third plane of '*apprenticeship*' focuses on a system of interpersonal involvements with individuals developing to become more responsible participants within a culturally organised activity. Although these planes are not separate, guided participation will be seen as central to the role of teachers in the classroom while the other roles will be more peripheral. This analysis moves away from a focus on the adult role, as seen in the work of Wood et al. (1976) and Tharpe and Gallimore (1988), to a recognition of the more interacting roles of adult and learner, and acknowledges the 'mutual involvement of individuals and their social partners' in the learning event. This mutual involvement is also reflected in the behaviours identified by Rogoff, Mistry, Goncu and Mosier (1993) in their study of the interactions of toddlers and adults where two distinct patterns of behaviour were noted. In one, the adult structured the children's learning by organising their attention, motivation and involvement and providing lessons from the ongoing activity, much in the spirit of the

Wood et al. (op cit) classification. In the other, children took primary responsibility for learning by managing their own attention, motivation and participation, with adults providing more responsive (than directive) assistance. The latter is mostly associated with home learning, while the former typifies many classroom practices.

Implementing Scaffolding in the Classroom

These studies above have implications for the classroom, but analysing adult/child interactions is not altogether adequate to account for the more specialised teacher/learner interactions that are relevant to mathematics learning. Taking consideration of the social dimension in learning, but this time based on classroom observations, Wood (1994) proposes two distinct patterns of interactions specifically observed in mathematics lessons. In the *funnel* pattern of interactions, students are provided with leading questions in an attempt to guide them to a predetermined solution procedure. This results in students needing ‘only to generate superficial procedures rather than meaningful mathematical strategies’. In contrast, the *focusing* pattern of interaction draws students’ attention to the critical aspects of a problem with the teacher posing questions to ‘turn the discussion back’, leaving responsibility for resolving the situation with the students (Wood, 1994, p. 155). The role of the teacher in supporting mathematics learning is one of summarising what is thought to be shared knowledge, and focusing joint attention on a critical point not yet understood.

In other studies that look specifically at classroom practices, but this time related to reading instruction, Hobsbaum, Peters, and Sylva (1996) make distinctions between *incidental scaffolding* – building on the child’s own overt intention within a shared, functional learning environment (as when a parent assists a child), and *strategic scaffolding* – adult deliberately teaches strategies which will enable the child to solve problems posed by a task (as related to lesson planning and the classroom). Analysing the effective supporting strategies of a Reading Recovery approach, the following key elements are identified:

- a measured amount of support without reducing the child’s initiative;
- careful selection of the task at just the right level of difficulty with the right balance of general ease but some challenge;
- the child must be able to make sense of the task using every available source of information;
- strategies made explicit – ‘this way of drawing explicit attention to strategies and processes provides a model of behavioural

regulation for the learner, which may become internalised, a 'voice in the head' for future situation. (Hobsbaum et al., 1996, p. 22).

Although these different elements were evident, in their observations Hobsbaum et al. (1996) still found the predominant teacher strategy, by a long margin, was 'telling' (p. 26). This suggests that more support is needed to help teachers reflect on what are effective interactions. Bliss, Askew, and Macrae (1996), in studying classroom teaching sequences in mathematics, science, and design and technology, looked for instances of scaffolding but also reported 'a relative absence of scaffolding in most lessons'. They identify '*actual scaffolds*' – approval, encouragement, structuring work, and organising people; '*prop scaffolds*' – where the teacher provides a suggestion that will help pupils throughout the task, and *'localised scaffolds*' – providing specific help 'where a teacher finds it difficult to help the pupil with an overall idea or concept simply because it is too large and complex'. They suggest two further scaffolds that are 'really more like cueing' and relate to the *funnel* pattern of interaction, namely:

- step-by-step or foothold scaffolds (often in a series of questions);
- hints and slots scaffolds (narrowing questions until only one answer fits). (Bliss et al., 1996) The latter two studies both suggest that scaffolding can take place most easily in a one to one teaching situation. In whole class teaching, meaningful interactions are more complex as contingent responding requires a detailed understanding of the learner's history, together with purposes of the immediate task and the teaching strategies needed to move individuals on.

TOWARDS A HIERARCHY OF SCAFFOLDING PRACTICES FOR MATHEMATICS LEARNING

Much of the background research on scaffolding has been drawn from studies that do not relate specifically to the mathematics classroom. The sociocultural approach of Rogoff (1995) has been helpful in analysing an activity or event on three different planes that are interdependent but which can each be made the focus for studies that can inform classroom practice. In a similar way, the following discussion will propose levels of scaffolding that can be found explicitly supporting mathematics learning with a range of contributory practices. The intention is to build on existing studies and to identify classroom practices that relate to mathematics teaching. By close analysis of observed interactions, attention is drawn to practices that good teachers often implement subconsciously, but that may be difficult for an inexperienced practitioner to identify. By articulating constituent elements in a hierarchy of practices, non-technical, professional language will be introduced that can be used to describe and reflect upon the actual act of teaching mathematics.

This paper will bring together support strategies from the aforementioned studies, together with new categories for mathematics learning, in particular *reviewing* and *restructuring*, that characterise effective teacher/learner interactions. Unlike Rogoff's (1995) planes of analysis that co-exist, the three levels for scaffolding proposed in this paper constitute a range of effective teaching strategies that may or may not be evident in the classroom.

At the most basic level, *environmental provisions* enable learning to take place without the direct intervention of the teacher. The subsequent two levels identify teacher interactions that are increasingly directed to developing richness in the support of mathematical learning through *explaining*, *reviewing and restructuring* and *developing conceptual thinking*. The diagram in Figure 1 shows a hierarchy that is structured to include observed patterns of interaction, with the central elements representing those most commonly seen, though not necessarily those that provides the strongest scaffolds. Those patterns represented peripherally are further supporting strategies that may be observed in the most effective mathematics teaching. The establishment of practices at different levels reflects not only the progressive (and often circular) supporting strategies that can be used, with each element having potential richness when it is given appropriate attention, but also the way effective interactions may be developed or, in some teaching, bypassed altogether.

Examples to illustrate the meanings at each level are taken from classroom observations and particularly from studies of geometry teaching in the early years of schooling and arithmetic in the later years, details of which are published elsewhere. In one such study, building blocks were used by groups of 4–6 year olds who were videotaped attempting specially designed tasks (Anghileri & Baron, 1998). An extension study with children of a similar age analysed the effectiveness of different adult interactions (Coltman, Anghileri, & Petyaeva, 2002). Research studies in arithmetic included video taped interviews with 9–11 year olds and audiotaped observations of 11–13 year olds in discussion with their teacher (Anghileri, 1995).

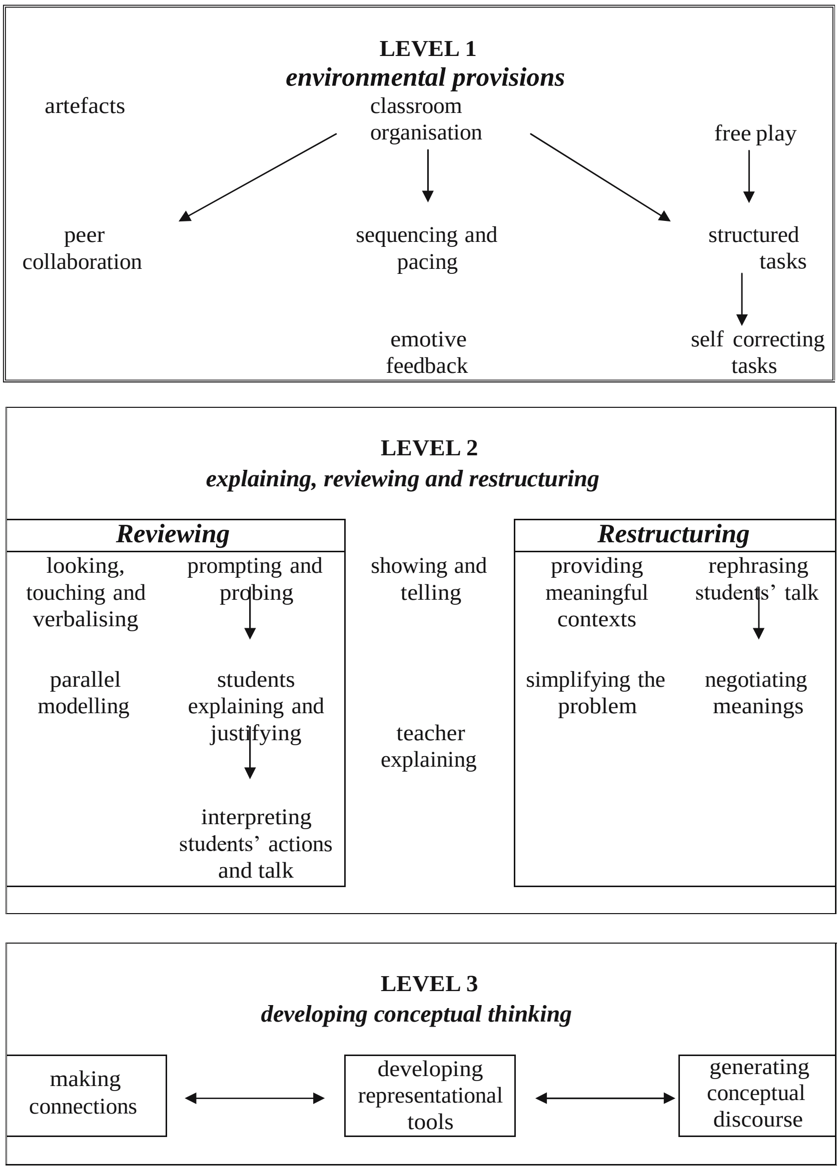

Figure 1. Teacher Strategies for Scaffolding Learning.

Level 1 Scaffolding

Before interacting with their students, teachers scaffold learning with *environmental provisions* including *artefacts* (for example, choice of wall displays, manipulatives, puzzles, appropriate tools) and *classroom* *organisation*, involving not only seating arrangements but also *sequencing* and *pacing* events. These need little introduction but are not always explicitly acknowledged as scaffolding (for example, Bliss et al., 1996) although they can have a significant impact on learning. At level 1, with appropriate provisions, learning can take place through interactions with *artefacts* in a classroom such as wall charts, puzzles and measuring apparatus. *Structured tasks* are most frequently provided as worksheets or directed activities. However, children using building blocks in *free play*, were observed to set their own challenges and learn through feedback, whether their attempts were successful or unsuccessful, with resulting improved performance on geometry tasks (Coltman et al., 2002). Modifying tasks to include a *self-correcting* element can provide further feedback that supports pupils' autonomous learning, not only in finding a solution, but also in reflecting on the processes involved in such a solution. Packing blocks into a constructed 'frame', with completion possible only for specific selections, meant success could be the result of persistent efforts (Coltman et al., 2002). In arithmetic, less automatic self-correction may occur in looking up the answer, or in practices like re-calculating by reversing the operation. This aspect of feedback can be found in some carefully prepared software packages that encourage reflection as well as offering corrections to the student.

Another strand of *environmental provision* involves grouping, so that learning can take place through *peer collaboration*, with students acting together to solve particular problems. Light and Littleton (1999) report 'compelling evidence for the benefits in terms of learning of peer collaboration'. Describing learning as the 'co-construction of understanding' they suggest that, through such work, progress appears to be associated with 'socially mediated processes of *conflict resolution*' (Light & Littleton, 1999, p. 91).

The scaffolding practices identified so far do not involve direct interactions between the teacher and students. There is, however, *emotive feedback* that will be included at level 1 where this does not directly relate to the mathematics to be learned. This includes the interjection of remarks and actions to gain attention, encourage, and approve student activities, each having a different quality from those that will be considered at level 2. It has been found that 'approval (and) encouragement' constitute the majority of interactions classified as 'actual scaffolds', along with 'structuring work' and 'organising people' (Bliss et al., 1996: 47).

Level 2 Scaffolding

At the next level, *explaining*, *reviewing and restructuring* involve direct interactions between teacher and students related specifically to the mathematics being considered. Figure 1 shows as central the tradition of *showing and telling* or *explaining* the ideas to be learned, and this relates to Wood's *funnel* stance, while the categories of *reviewing* and *restructuring* identify patterns of interaction that are more responsive to the learner and these expand on the idea of *focusing* (Wood, 1994).

Showing and telling have been traditional in classroom teaching for generations and continue to dominate classroom practice (Hobsbaum et al., 1996; Pimm, 1987). With this strategy, teachers retained control and structured conversations to take account of the 'next step' they have planned, with little use being made of the pupils' contributions. Equally one-sided can be the *explaining* that is often satisfying for a teacher while inadvertently constraining students' thinking – acting as a kind of closure to discussion. Where the explanation is not 'in tune' with a student's thinking this can compound the difficulty, giving the student a problem in reconciling different ideas (Anghileri, 1995).

Alternatives to showing and telling involve developing students' own understanding of mathematics through *reviewing* and *restructuring*. The former relates to interactions where the teacher encourages experiences to focus students' attention on pertinent aspects of the mathematics involved. The latter involves teachers making adaptations to modify the experiences and bring the mathematics involved closer to students' existing understanding.

Reviewing

When students are engaged with a task, they are not always able to identify those aspects most pertinent to the implicit mathematical ideas or problem to be solved. A response for teachers is to refocus their attention and give them a further opportunity to develop their own understanding rather than relying on that of the teacher. *Reviewing* classifies five such types of interaction:

- getting students to *look, touch and verbalise* what they see and think;
- getting students to *explain and justify*.
- *interpreting students' actions* and talk;
- using *prompting and probing* questions and
- *parallel modelling*.

Enabling students to develop their own meanings in these ways can have long-term benefits in enhancing their confidence and independence in learning.

Looking, Touching and Verbalising

Looking, touching and verbalising can bring different senses to bear on a problem by encouraging students to handle manipulatives, reflect on what they can see, and repeat instructions or verbalise observations. The teacher's encouragement to ''pick up a block, hold it with both hands, turn it around and look at it carefully'' was successful in helping young children in a task of matching 3D shapes to their 2D faces (Coltman et al., 2002). When unable to continue a repeating sequence of three blocks s/he had seen being constructed, the teacher asked the child to ''tell me the colours'' as the sequence was built. The resulting reflection enabled the child to continue correctly. Similarly, in arithmetic teaching, the encouragement to ''tell me what you did'' will frequently lead a student to verbalise their thinking and notice an error in reasoning or in calculating that they can correct for themselves. 'It seems that the act of attempting to express their thoughts aloud in words has helped pupils to clarify and organise the thoughts themselves' (Pimm, 1987, p. 23).

Prompting and Probing

Unlike parent–child interactions in which the child will often take the initiative, classroom interchanges often revolve around *prompting* questions that involve pupils in trying to guess what response the teacher is looking for, rather than giving their personal thoughts. Using 'clozed' questions that require one word answers generates an I(nitiation) – R(esponse) – F(eedback) framework which locks the teacher into 'centre stage', as controller of communication (Pimm, 1987). Such questions can cause a teacher to make unwarranted assumptions about children's understanding 'because the children are merely picking up on cues from the questions themselves' (Tharpe & Gallimore, 1988, p. 232). Wood provides detailed analysis of classroom exchanges to illustrate a *funnel pattern* of interactions with *prompting* questions that successively lead the students towards a predetermined solution (Wood, 1994). Such questioning can be valuable in supporting a students' thinking but needs teachers to be responsive to the students' intentions rather than their own. *Probing* questions on the other hand will try to get the students to expand on their own thinking. It is the role of the teacher to interject questions that focus on the most critical points in an explanation and take the understanding forward. Here the purpose is to gain insight into students' thinking, promoting their autonomy and underpinning the mathematical understanding that is generated.

Interpreting Students' Actions and Talk

It is noted that 'the learner must be able to *recognise* a solution to a particular class of problems before he is himself able to produce the steps leading to it without assistance' (Wood et al., 1976, p. 90). An example from geometry is given where children built a brick tower to match a given height. After many experimental attempts, the teacher interacted to *make explicit* the relevant action: ''Well done, you turned the triangular prism to make a tall column''. This comment drew attention to the most pertinent aspect of the child's construction and at the same time provided language that would, ultimately, facilitate the child's reflections on the task (Coltman et al., 2002). In arithmetic, teachers can identify steps involved in solving a problem, focusing on progressive development of pupils' own strategies rather than 'over writing' the student's approach with a formal algorithm (Anghileri, 2001). Where students share their strategies it is sometimes necessary for the teacher to expand on the explanation of an individual to make explicit the key characteristics of a solution. The strategy for calculating 6 + 7 as ''6 add 6 and one more'' was made more explicit by the teacher who expanded to ''Jan knew that the answer to 6 + 6 is 12 and knew that 6 + 7 would be one more, which is 13''.

Parallel Modelling

When the reflective interactions identified above are not sufficient to lead to the solution of a problem, there can be a temptation to 'show' or 'tell' a solution, but an alternative strategy is *parallel modelling* (Coltman et al., 2002). Here the teacher creates and solves a task that shares some of the characteristics of the student's problem. Using a long thin cuboid, rather than the long triangular prism, as a column for building the tower referred to in the last section, will complete the task using a different block with similar properties. The student retains ownership of the original task but has the opportunity to see a parallel task being solved and to transfer understanding. This is often used in arithmetic teaching with the provision of 'worked examples', although it may not always be clear which examples will trigger a particular solution. Different choices of numbers may not provide a parallel type of calculation. The calculations $140 \div 5$ and $140 \div 4$ involve close numbers, but mental solutions could involve division by 10 and doubling in the first case and repeated halving in the second case. It is difficult to identifying a number of calculations to be solved using the same method without limiting students' appropriate choice of strategy.

Students Explaining and Justifying

In contrast to teaching built upon teachers' explanations, social norms can be established in the classroom where the students themselves are expected to go beyond simply verbalising, such as repeating instructions or describing a situation, to *explain and justify* their solutions (Cobb, Wood, & Yackel, 1991). The role of the teacher is to promote mathematical understandings through the 'orchestration' of small group and whole class discussions where students actively participate by making explicit their thinking, by listening to contributions made by classmates and indicating when they do not understand an explanation, and by asking clarifying questions. This will also help the teacher to monitor the understanding of individuals. For example the solution to $6 + 7$ was achieved mentally but with different explanations:

''6, 7, 8, 9, 10, 11, 12, 13''
''7, and 3 makes 10 and 3 more makes 13''
''6 add 6 and one more is 13''

Not only did explaining their own strategies and listening to those of others help the students, but the act of individuals justifying their approaches to different tasks appeared to promote reflective thinking. Through such elaborations, teachers will also be better informed of each individual's mathematical understanding and this will facilitate teaching that is responsive to the needs of these individuals.

Restructuring

Through *restructuring*, the teacher's intention is progressively to introduce modifications that will make ideas more accessible, not only establishing contact with students' existing understanding but taking meanings forward. This differs from *reviewing* where teacher–student interactions are intended to encourage reflection, clarifying but not altering students' existing understandings. Restructuring involves interactions such as:

- provision of *meaningful contexts* to abstract situations;
- *simplifying the problem* by constraining and limiting the degrees of freedom;
- *rephrasing students' talk* and
- *negotiating meanings*.

Identifying Meaningful Contexts

Where students cannot solve an abstract problem, a context can help students identify something within their experience that is related. In arithmetic, the shift from an abstract calculation: '6÷ 12¼'— to a contextual setting: ''Six pizzas to be shared among 12 people.'' took a problem from inaccessibility to the construction of a meaningful solution (Anghileri, 1995). Such an approach revolves around imagery and scaffolding students' thinking involves ensuring that (their) activity remains grounded in the mathematical imagery of the situation (Cobb et al., 2000). Imagination is powerful, as illustrated where success on a construction task with geometric blocks was markedly improved when a 'story' was given. The teacher named groups of three blocks as 'mother elephant, baby elephant and keeper' and placed them in a 'carriage of a jungle train' to give meaning to a repeating sequence task. Success on the contextual 'elephant' task was followed by success on post-test, abstract sequences (Coltman et al., 2002). Acknowledging that mathematical understanding must extend to applications of abstract concepts and processes, the move to abstraction will need to be progressive, and may require the introduction of a number of different contexts. Students will come to identify key characteristics of a problem and relate them to familiar contexts as they develop their own understanding of the abstract links.

Simplifying the Problem

Where a student is unsuccessful, it is sometimes possible for the teacher to *simplify a task* so that understanding can be built in progressive steps towards the larger problem. This can be identified with reduction in degrees of freedom, in order to establish contact with the students' existing understandings, so that any given feedback is regulated to a level which could be used for correction, (Wood et al., 1976). In the geometry studies identified above, one task involving continuation of a sequence of blocks (e.g. red cube, blue cuboid, yellow cone, red cube, blue cuboid, ...). The teacher simplified this to a repeating sequence of only two shapes, or a single shape (Coltman et al., 2002). Both adaptations were introduced to make more explicit the nature of the repeating sequence before additional characteristics were re-introduced for more complex tasks.

Re-phrasing Students' Talk

Re-phrasing students' talk is an important role of the teacher for highlighting processes involved in solutions, re-describing students' efforts to make clear the mathematical aspects that are most valued. Considerable sensitivity may be needed to 'unpick' the essence of students' talk, rephrasing where necessary to make ideas clearer without losing the intended meaning, and negotiating new meanings to establish mathematically valid understandings (Anghileri, 1995). This clearly relates to the strategy of *interpreting students' actions and talk* of the previous section but goes further by introducing and extending the more formal language of mathematics. The vocabulary children use can also be inaccurate and teachers can introduce correct terminology to re-phrase the student's intent. For example, the teacher used the correct term 'cube' to describe blocks named as 'squares' by children talking about their activities (Coltman et al., 2002). Using formal mathematical terminology is only part of the development in children's learning as language is not a means of transporting conceptual structures from teacher to student, but it is an element of interactions that allows the teacher to constrain and to guide the cognitive construction of the student.

Negotiating Meanings

As a teacher pays close attention to pupils' talk, many 'spoken formulations and revisions will often be required before an acceptable and stable expression can be agreed upon by all participants' (Pimm, 1987, p. 23). This process of *negotiating meanings* involves a 'social process of developing a topic, by pooling and probing predicates and by selecting socially agreed-on predicates' as classroom discussion becomes 'the collective learning of the classroom community, during which taken-as-shared mathematical meanings emerge as the teacher and students negotiate interpretations and solutions' (Gravemeijer, Cobb, Bowers & Whitenack, 2000, p. 226). It is time consuming and demanding on a teacher's skills to elicit the true meanings of their students' responses, respecting the more outlandish contributions as their students work at developing their personal understanding, and not simply opting for responses that are 'in tune' with their requirements (Anghileri, 1995). Sometimes it is feared that students initiating incorrect meanings could spread misunderstanding but research has shown that learning improves where errors and misconceptions are exposed and discussed (Askew & Wiliam, 1995). It is through a struggle for shared meaning that a process of cooperatively figuring things out determines what can be said and understood by both teacher and students and this is what constitutes real mathematics learning in the classroom.

Level 3 Scaffolding

Mathematics learning involves more than the ability to replicate taught procedures and solve isolated problems. In mathematics there are particular needs as teachers are looking for the development of concepts through specialised processes such as generalisation, extrapolation and abstraction. It is here that the third level of scaffolding strategies becomes imperative. This highest level of scaffolding consists of teaching interactions that explicitly addresses *developing conceptual thinking* by creating opportunities to reveal understandings to pupils and teachers together. Such supports match most closely the *cognitive structuring* identified by Tharpe and Gallimore (1988) but are often lacking in classroom interactions. As pupils are supported in *making connections* and developing a range of *representational tools*, transferable skills and understanding that can be communicated become established. At this highest level, teachers in the classroom can engage their pupils in *conceptual discourse* that extends their thinking. Once again, in the hierarchy (Figure 1) the establishment of *representational tools* is located centrally as it is most commonly found where teachers notate mathematical processes. Less commonly found, but identified as the most effective interactions, are those specifically focused on *making connections* and *generating conceptual discourse*.

Developing Representational Tools

Much of mathematical learning relates to the interpretation and use of systems of images, words and symbols that are integral to mathematical reasoning. 'Mathematics as a discipline is now generally conceived as an activity in which constructive representation, with the help of symbols, plays a decisive role' (van Oers, 2000). In understanding number, words and symbols become inextricably linked for notating thinking and for organising thinking itself. In working with practical tasks the *representational tools* can revolve around language, both the informal language that evokes images familiar to the children (for example, a triangular prism recognised as the 'roof' shape), and the formal names (triangular prism) that children begin to use as they refine their understanding of particular characteristics (Coltman et al., 2002). Representations also include the structuring of practical activities to provide powerful visual imagery. Sorting a set of dominoes, for example, uses the objects themselves to create a representation of the complex structure of a complete set (try this activity if you have not done so).

In addition to providing a means of communication through words and symbols, teachers can develop other representations as tools for structuring knowledge, for example in constructing and interpreting graphical representations and spreadsheets. The focus is not on the symbols themselves, but on the activity of meaning making for the mathematical structures represented.

With teacher guidance, a symbolic record can facilitate discussions, and representations can become tools for thinking. Teachers' scaffolding can involve 'notating students' interpretations and solutions'... so that these symbolisations would then constitute a resource that students can use to express, communicate, and reflect on their mathematical activity (Cobb et al., 2000). This will be the case, for example, in early work on addition and subtraction where different representations can be introduced to focus on the connection between the two operations that are not evident in the formal symbolisation.

Making Connections

Teacher interventions appear to be a key to developing children's ideas in a connected way, and *making connections* is crucial as a strategy to support mathematics learning. This can be an extension of the restructuring strategies identified above as new associations can be introduced, for example, using the idea of 'doubling 6' instead of '6 add 6' in rephrasing the student's solution in the example above.

In a study of effective numeracy teaching, the term '*connectionist*' was used to describe approaches where emphasis is given to the links between different ideas in mathematics, and where pupils are encouraged to draw on their mathematical understanding to develop their own strategies in problem solving (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997). This research found that highly effective teachers believe that pupils develop strategies and networks of ideas by being challenged to explain their thinking and to listen to the thinking of others. Such teaching approaches build on pupils' own strategies with teacher interventions to clarify the thinking, and make explicit the aspects that are most critical to understanding (Wood, 1994).

Learning of decimals gives an example of the way learning can be enhanced if connections are made with fractions and percentages:

'if they (children) know that 1/2, 0.5 and 50% are all ways of representing the same part of a whole, then the calculations

$$1=2 \times 40$$
$$40 \times 0:5$$
$$50\% \text{ of } 40$$

can be seen as different versions of the same calculation.' (QCA, 1999: 52) There is evidence that lack of connections, for example between students' informal approaches and taught procedures, can result in little progress, while teaching approaches that progressively develop connections lead to better understanding (Anghileri, 2000).

Generating Conceptual Discourse

Within *conceptual discourse*, the teacher goes beyond the explanations and justifications of Level Two Scaffolding by initiating reflective shifts such that what is said and done in action subsequently becomes an explicit topic of discussion (Cobb, Boufi, McClain, & Whitenack, 1997; Wood, 1994). For example, after sorting shapes to select one that will roll, the teacher asked 'Why will it roll?' and the ensuing discussion encapsulated many observed features of mathematical value about the concept of curved surfaces. With such a conceptual orientation, students are likely to engage in longer, more meaningful discussions, and understanding comes to be shared as the individuals engage in the communal act of making mathematical meanings. The example of different students' solutions for solving $6 + 7$ (used earlier in this article) could also be developed by talking about the strategy of doubling and using near doubles that has applications in a whole class of problems, perhaps getting students to suggest their own examples. While accepting a wide range of students' explanations, teachers can indicate thinking strategies that are particularly valued, thus enabling students to become aware of more sophisticated forms of mathematical reasoning. Teachers play a vital role in shaping this discourse through signals they send about the knowledge and ways of thinking and knowing that are valued. McClain, Cobb, Gravemeijer, and Estes (1999) identify *conceptual discourse* as central in developing mathematical thinking as it makes possible the students' development of mathematical beliefs and values that contribute to their development of intellectual autonomy. Two characteristics of classroom discourse that relate specifically to mathematics learning are the norms and standards for what counts as acceptable mathematical explanation (conceptual not computational), and the content of the whole class discussion (Cobb et al., 1991).

CONCLUSION

The purpose of this article has been to identify and classify classroom interactions that can be effective for mathematics learning. Current paradigms for learning suggest that teachers will be most effective if they are able to scaffold pupils' learning by employing a range of teaching approaches in an environment that encourages active involvement. This article has attempted to provide fine detail on the alternative strategies that are available. All the levels of scaffolding identified are possible, from the provision of tasks and resources, to the engagement in conceptual discourse, and the proposed analysis is developed to support the practitioner in reflection and analysis of actual classroom practices. When it is recognised that some teaching provides only minimal support, for example textbook exercises that are set and marked, with explanations given by a teacher, it becomes possible to introduce enriching practices through the extended range of interactions outlined above.

And what of the appropriateness of scaffolding as a metaphor for supporting mathematical learning? With the most literal interpretation, scaffolding refers to 'bolted together tiers of boards upon which human workers stand to construct a building' and this analogy admits 'more easily of variation in amount than in kind' (Rogoff & Wertsch, 1984, p. 47). This rigid structure that precedes the central building is analogous with some instructional practices. Indeed, this reflects how instruction in mathematics has been perceived in the past, where children were trained in standardised procedures of arithmetic and geometry and acculturated into historical practices. Even today 'teaching in the United States (and many parts of the world) tends to be a highly routinised activity that is scripted in advance and involves few adaptations to students' contributions' (Cobb et al., 2000).

The notion of scaffolding also presupposes that learning is hierarchical and built on firm foundations, while teachers know that elements of understanding can appear in students as an eclectic collection until connections are established. Application to isolated tasks, with the steady withdrawal of support and establishing of independence, also has shortcomings in the broader context of schooling where the learner is continually pressed to achieve beyond individual tasks and to extend understanding.

With the identified weakness of scaffolding as an image of the teaching process, what is needed for a metaphor of classroom practice is perhaps the notion of a flexible and moving scaffold (that allows for individual creativity) in which teachers are responsive to individuals even within the classroom setting. The goal in teaching is for autonomous and independent, self-motivated learners. For this purpose, flexible and dynamic scaffolding will need to be responsive to the emerging learner within the social group.

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